Session 2 Essay Assignments

Mohammad Keyhani

Instructor: Behrang Koushavand

# Write an Essay to explain the concept of expected value, with an example.

When a measure or variable can take on multiple values each time it is observed, recorded or measured, and the occurrence of those values involves randomness with known probabilities, then we know the probability distribution of that variable. Then the expected value of the variable is defined as the weighted average of all the values that the variable can possibly take on, weighted by the probability of each of those values.

In the case of a uniform probability distribution where the probability of all values is equal, the weights in the weighted average all become equal and the weighted average becomes equivalent to the regular arithmetic mean of the values. Even when the probability of each value is not equal, the expected value and the arithmetic mean have a close relationship: with large enough number of observations, the simple arithmetic mean of all observations converges to the expected value of the probability distribution.

The larger the variance of a probability distribution, the more any single observation can deviate from the expected value. Thus when variance is larger, more observations are required for the realized expected value of the observations (sample) to approach the actual expected value of the population calculated from the probability distribution.

For example, the height of people in a population usually has a normal (Guassian) or bell curve probability distribution, where most people are around average height, and the more extreme short heights and extreme long heights are rare.

For example between a population of 7 adults you may observe the following heights:

170 cm: 3 men

150 cm: 2 men

190 cm: 2 men

195 cm: 1 man

145 cm: 1 man

For this population, the probability of each height value occurring, is the proportion of the people with that height value (the number of people with that height value divided by the total number of people, 9). So the expected value is:

170\*(3/9)+150\*(2/9)+190\*(2/9)+195\*(1/9)+145\*(1/9)= 170

You can see in this case that the expected value is equal to the arithmetic mean of the population as well. The mean is calculated as:

170\*(1/9)+170\*(1/9) +170\*(1/9)+150\*(1/9)+150\*(1/9)+190\*(1/9)+190\*(1/9)+195\*(1/9)+145\*(1/9)=170

# Write down the equation for mean, variance, and standard deviation.

If each observation of a population of is denoted by then the equation for arithmetic mean is as follows:

The equation for variance of a population is:

Note: if we are calculating sample variance instead of population variance, we use as the demoninator rather than due to Bessel’s correction[[1]](#footnote-1) in order to get an unbiased estimator of the population variance.

The equation for standard deviation of a population is the square root of the variance:

Which again, with Bessel’s correction, would be:

# What is the difference between a sample and a population?

A population is the entire set of possible observations of a random variable. Usually not all possible observations are observed or measured and only a sub-population of them are actually observed. We call this sub-population a sample when we want to estimate the parameters of the population using only information from the sample. We usually try to collect the sample observations through a fully random process, so that they are as representative of the broader population as possible.

# What is the average of a continuous variable for a population?

The mean of a continuous random variable for a population can be calculated if we know the probability density function for the population over all possible values of the random variable of interest. Since the variable is continuous, instead of summing like with discrete variables, we need to integrate. The formula for the mean is as follows:

1. <https://towardsdatascience.com/why-sample-variance-is-divided-by-n-1-89821b83ef6d> [↑](#footnote-ref-1)